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A mathematical optimization framework for expansion draft decision making and analysis

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In this paper, we present and analyze a mathematical programming approach to expansion draft optimization in the context of the 2017 NHL expansion draft involving the Vegas Golden Knights, noting that this approach can be generalized to future NHL expansions and to those in other sports leagues. In particular, we present a novel mathematical optimization approach, consisting of two models, to optimize expansion draft protection and selection decisions made by the various teams. We use this approach to investigate a number of expansion draft scenarios, including the impact of collaboration between existing teams, the trade-off between team performance and salary cap flexibility, as well as opportunities for Vegas to take advantage of side agreements in a leverage experiment. Finally, we compare the output of our approach to what actually happened in the expansion draft, noting both similarities and discrepancies between our solutions and the actual outcomes. Overall, we believe our framework serves as a promising foundation for future expansion draft research and decision-making in hockey and in other sports.

Key words: Operations Research, Mathematical Programming, Optimization, National Hockey League, Expansion Draft

1. Introduction

In 2017, the Vegas Golden Knights joined the National Hockey League (NHL) as the first expansion team since the Columbus Blue Jackets and the Minnesota Wild joined in 2000. As newly introduced teams to the league, expansion teams have the unique opportunity to draft active, NHL-ready players directly from their opponents – essentially roster optimization from a clean slate with a salary cap. Existing teams can mitigate their loss by carefully choosing which players are exposed for selection and by applying smart roster management leading up to the expansion draft.

Expansion drafts in professional sports occur when a league decides to add new teams to its current pool of teams. These drafts have been used to grow leagues across a variety of sports, including hockey, American football, baseball, and basketball. Although the detailed rules of an expansion draft may differ across sports, its structure remains largely consistent, involving: a *protection* process where existing teams identify players that they wish to shield from being selected

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by the expansion team, and a *selection* process where the expansion team selects players from some or all of the existing teams to fill its roster. Constraints may be imposed in each of these processes, such as limits on the number of players that can be protected by a team, and positional requirements that must be fulfilled by the expansion team's selections. Together, the rules governing who can be protected and who can be selected form part of a decision making problem that is amenable to rigorous, mathematical optimization. Thus, league expansion represents a perfect opportunity to apply optimization in sports team management.

In this paper, we present and analyze a mathematical programming approach to expansion draft optimization in the context of the 2017 NHL expansion draft involving the Vegas Golden Knights, noting that this approach can be generalized to future NHL expansions and to those in other sports leagues. We develop two optimization models: the first formulation models the existing teams' protection problem and the second formulation models the expansion team's selection problem. The first model can be used by each existing team to optimize the players it protects (or equivalently, exposes) in the draft. The second model can be used by the expansion team to identify a salary cap-compliant team of players given a list of exposed players by the existing teams. Together, these models form a novel framework to aid the expansion team in evaluating and optimizing against different exposure scenarios it may encounter during the expansion draft.

In the case of the 2017 NHL expansion draft, the expansion team, Vegas, had a short turnaround time – three days from when the existing teams finalized their protection lists to when Vegas had to make their selections on June 21, 2017 – amplifying the value of our optimization framework, which can evaluate a large number of possible selections very quickly. Indeed, Craig Button, a former NHL general manager and a current broadcast analyst, reiterated "Who you decide to select from one team has serial impact on what you do with every other one. There are lots of combinations and permutations" (Seravalli (2016b)). Existing teams can also use this framework to guide their protection decisions and other personnel decisions leading up to the expansion draft such as trading players with other teams or signing free agents. Our model provides structure to the vast combinatorial decisions faced by the expansion team, which an existing team can use to optimize their exposure list accordingly.

After presenting our optimization framework, we conduct several computational experiments aimed at shedding light on the value of mathematical optimization in the context of the 2017 NHL expansion draft. First, our baseline experiment tracks the quality of the team Vegas could have drafted, given optimized protection lists of the other 30 teams, at four time points over the course of the 2016-17 season. Second, we illustrate how our model can be used to generate rosters for Vegas that trade off between performance and salary cap flexibility. Throughout, we use the Point Shares metric as a measure of player value/performance, although our framework is general

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enough to accommodate any statistic. Third, we show how our framework can tractably investigate an important and realistic scenario that can hurt Vegas leading up to the expansion draft. This "collaboration" experiment refers to teams making personnel decisions such as trades that would minimize their exposure and the value of assets lost in the expansion draft. Indeed, Vegas' general manager George McPhee said he expected a "redistribution of players" to occur ahead of the draft (Seravalli (2016a)). For example, a team with valuable exposable assets could make a trade with another team with more protection room, in return for future draft picks or prospects who would be exempt from selection. Fourth, we conduct a series of experiments to show how optimization can support Vegas in making side deals with teams to avoid picking a particular player, but still create a team with high value. By identifying these opportunities to "leverage" other teams to provide incentives to not pick a particular player, Vegas can potentially extract extra assets such as draft picks or prospects, while still creating a good team overall by optimizing the other picks accordingly. Finally, we discuss the results of our experiments in the context of what actually occurred in the expansion draft.

Our main contributions are as follows:

1. We develop a novel optimization framework for expansion draft optimization, applied in particular to decisions faced by the existing teams and the expansion team under the 2017 NHL expansion draft rules. The framework is general and with simple modifications can be extended to future NHL expansion drafts, as well as those found in other sports with different expansion draft rules. Our selection model, with minor modifications, is also applicable to fantasy sports lineup optimization.

2. We use our optimization framework to quantify Vegas' team value under a variety of scenarios:

(a) We demonstrate that Vegas' potential team value degrades by over 13% over the course of the 2016-17 season leading up to the expansion draft as a result of personnel decisions made by the existing teams.

(b) By considering future financial flexibility, we show that Vegas can select a team that degrades in team value by only 4.9% but with a 57.1% reduction in 2018-19 salary commitments.

(c) Our collaboration experiment shows that coordinated personnel decisions made by other teams can have significant impact on Vegas' team quality. Our structured approach to investigating this concept shows that had teams focused on minimizing the value of their exposed assets to Vegas, they had the potential to degrade Vegas' team value by up to 34%.

(d) Our leverage experiment shows that the second best pick among the exposed players of any given team may still lead to a high quality team for Vegas, after re-optimizing the other 29 selections. For the top 10 teams that had the most to gain from Vegas not selecting the optimal player suggested by the framework, Vegas' team value only degraded by 1.9% on average, suggesting

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that Vegas had leverage to extract additional assets from those teams in return for not selecting the suggested player, while giving up little in terms of overall team value.

2. Literature

This paper touches on several aspects of the sports literature: hockey analytics, optimization in sports, and sports league expansion. We briefly review each area.

In the area of hockey analytics, most research has focused on developing advanced stats (Riley (2017), Macdonald (2012), Franks, DAmour, Cervone, and Bornn (2016), Shea and Baker (2012), Mason and Foster (2007), Macdonald (2011), Pettigrew (2015)), player classification and valuation methods, (Dawson and Magee (2001), Gramacy, Jensen, and Taddy (2013), Vincent and Eastman (2009), Chan, Cho, and Novati (2012), Kaplan, Mongeon, and Ryan (2014), Thomas, Ventura, Jensen, and Ma (2013), Schuckers and Curro (2015)), and in-game decision making (Schulte, Khademi, Gholami, Zhao, Javan, and Desaulniers (2017), Thomas (2006), Rigdon (2011)). Advanced statistics and player valuation metrics are applicable to expansion draft optimization since they can serve as coefficients in the model representating the "value" of the players. While there has been much work on in-game decision making in hockey, such as the rich literature on pulling the goalie (e.g., Washburn (1991), Beaudoin and Swartz (2010)), there has been less focus on research related to team management decisions.

Optimization methods have been applied to a variety of sports analytics problems across a range of sports including baseball (Chan and Fearing (2017)), soccer (Duran, Guajardo, and Saure (2017)), tennis (Chan and Singal (2016)), and motorcycle racing (Amoros, Escudero, Monge, Segura, and Reinoso (2011)). The most common applications tend to be in scheduling, such as scheduling sports leagues (Van Voorhis (2002)), tournaments (Duran et al. (2017)), and baseball umpires (Trick, Yildiz, and Yunes (2011)). Team formation is a natural application for optimization and typically formulated as a variation of the knapsack problem; the closest studies in the literature to ours include using optimization to choose fantasy football lineups (Becker and Sun (2016)) or soccer and volleyball teams (Boon and Sierksma (2003)). Dynamic programming has also been used to select players in a sports league entry draft (Fry, Lundberg, and Ohlmann (2007)). Uncertainty in performance of selected players in a team formation problem can be modeled using a stochastic knapsack problem (Gibson, Ohlmann, and Fry (2010), Pantuso (2017)). The use of mathematical optimization to determine lineups in the context of an expansion draft has not been studied before.

Finally, there has been some research on league expansion. Specifically for the NHL, there have been studies on realignment of the league structure (MacDonald and Pulleyblank (2014)) and the viability of potential new teams (Light, Chernin, and Heffernan (2016)). Additional studies have

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examined the impact of league expansion on competitive balance in baseball (Schmidt (2001), Quinn and Bursik (2007)) and the power dynamics between existing and expansion teams in football (Dickson, Arnold, and Chalip (2005)). Modeling the decision-making process of an existing or expanding team in the expansion draft process itself has not been considered previously.

3. Expansion Draft Background

In this section, we briefly summarize previous expansion drafts across a variety of sports leagues and then review the specific rules relevant to the 2017 NHL expansion draft. The complete set of official rules can be found on the NHL's website (NHL (2016b)). These rules are summarized as a set of protection constraints and a set of selection constraints, and can be applied to other expansion drafts by altering the parameter values and including new side constraints, as needed.

3.1. Previous Expansion Drafts

Expansion drafts have long been a method for growing sports leagues. In addition to the NHL, Major League Baseball (MLB), the National Basketball Association (NBA), and the National Football League (NFL) have all been holding expansion drafts since the early 1960s. These events are approached similarly across the various leagues; the expansion team is tasked with selecting a roster from the pool of players that are made available by the existing teams. In some circumstances, as in the NFL, the new team is also awarded a high-ranking college draft pick (usually first or second) to further ensure their competitiveness. In this paper, we solely focus on the expansion draft process.

Initially, these processes were used for ambitious sports league growth, as illustrated by the 1967 NHL, 1968 MLB, and 1970 NBA expansion drafts, with six, four, and three expansion teams introduced at the same time, respectively. Due to the small size of existing team pools at the time, these events had unique protection/selection requirements that saw existing teams lose a significant portion of their rosters. As sports league growth tapered, more recent expansion drafts have introduced one or two new teams at a time. Indeed, three out of the four NHL expansion drafts held since 1998 involved a single expansion team, including the 2017 draft. Furthermore, the larger pool of existing teams in recent drafts has resulted in new teams typically selecting a single player from each existing roster.

Throughout the years, expansion draft rules have been revised to accommodate new constraints aimed at producing a more equitable outcome. Examples include the addition of team salary cap requirements (NHL, NFL, and NBA), restrictions on the movement of young professionals or unrestricted free agents (NHL and NFL), and position-specific selection/protection limits (MLB, NHL, and NFL). These requirements are incorporated into our framework as side constraints within

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either the selection or protection model, and can be readily revised to align the modeling with new rules. As an illustrative example, the 2000 NHL expansion draft allowed existing teams to protect either 9 forwards, 5 defenseman, and 1 goalie, or 7 forwards, 3 defenseman, and 2 goalies. This rule was amended for the 2017 expansion draft to improve the competitive viability of the expansion team. Under the new rule, existing teams could protect either 7 forwards, 3 defenseman and 1 goalie, or 8 skaters (forwards/defenseman) and 1 goalie; such an adjustment is easily implemented within our framework.

3.2. The 2017 NHL Expansion Draft

The 2017 NHL expansion team, Vegas, creates its roster by selecting from a pool of players that are made available by the existing 30 teams. Following previous expansion drafts in the NHL and other sports leagues, the rules are specified with respect to the existing teams and the expansion team.

3.2.1. Rules for Existing Teams Existing teams must "protect" a certain number of players from their roster. A protected player cannot be selected by Vegas. Each existing team must:

P1. Protect one goaltender (G)

P2. Protect either seven forwards (F) and three defensemen (D) or eight total skaters (F and D combined)

P3. Protect all players with no movement clauses (who do not waive their clause), counting them towards the protection quotas

P4. Retain all first- and second-year professional players and unsigned draft choices; they are exempt from selection and do not count towards protection quotas

Players who are not protected are "exposed". These are the players that Vegas can choose to form their roster. Each of the 30 teams must expose at least:

E1. One defenseman under contract for 2017-18 who played 40+ NHL games in 2016-17 or 70+ NHL games in 2015-16 and 2016-17 combined

E2. Two forwards under contract for 2017-18 who played 40+ NHL games in 2016-17 or 70+ NHL games in 2015-16 and 2016-17 combined

E3. One goaltender under contract for 2017-18 or who will be a restricted free agent in summer 2017 (and has received a qualifying offer by the existing team)

3.2.2. Rules for Expansion Team Once each existing team has finalized its list of protected players, the expansion team, Vegas, must adhere to the following rules regarding selections:

S1. Select exactly one exposed player from each of the 30 existing teams

S2. Select at least 14 forwards, nine defensemen, and three goaltenders

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S3. Select at least 20 players who are under contract for the 2017-18 season, of which at least 18 should be skaters and at least two should be goaltenders

S4. Select a team with aggregate salary (based on salary cap) of 60-100% of the 2016-17 season salary cap, equivalent to \$43.8M-\$73.0M

S5. Players selected cannot be bought out before summer 2018

Regarding the third selection rule, S3, the official rules on the NHL expansion draft website do not explicitly state that at least 18 need to be skaters and two need to be goaltenders; we inferred these more granular requirements from the Hockey Operations Guidelines (NHL (2016a)), which describe the size and composition requirements of opening day rosters.

4. Optimization Models

In this section, we present our optimization models for player protection (existing teams) and player selection (expansion team). The first few subsections translate the rules from Section 3 into mathematical constraints for the two models, describe how side deals between the expansion and existing teams can be modeled, and describe the objective functions for the models. The last two subsections provide details on the data and model implementation, respectively. Our models are designed to capture the constraints relevant to the 2017 NHL expansion draft, as summarized in Section 3.2, but can easily be modified to accommodate future expansion draft rules, as previously noted.

4.1. Protection Model Constraints

Let I denote the set of all players. The protection model's (binary) decision variable is y_{it}, defined to be 1 if player $i \in I$ from team t is protected, and 0 otherwise. Let T denote the set of all teams. Let I_f^t, I_d^t, and I_g^t be the set of non-exempt forwards, defensemen, and goalies, respectively, from team $t \in T$. Recall from Section 3.2.1 that first- and second-year professional players, as well as unsigned draft choices are exempt from the expansion draft, and thus do not count towards the protection quotas. Let $I_{E_f}^t$, $I_{E_d}^t$, and $I_{E_g}^t$ be defined similarly, but comprise players who meet the exposure requirements for their respective positions. Let I_p^t be the set of players from team $t \in T$ who must be protected (players with no movement clauses). We also let $n_g^p, n_f^p, n_d^p, n_{f\cup d}^p$ represent the number of goalies, forwards, defenseman, and skaters that must be protected and n_g^e, n_f^e, n_d^e the number of those who meet the specified exposure requirements to be exposed, according to the draft rules from the previous section. With this notation, we can now translate the protection rules for each team $t \in T$ into mathematical constraints, as below.

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$$\sum_{i \in I_g^t} y_{it} = n_g^p, \tag{1}$$

$$\left.\begin{aligned} \sum_{i \in I_f^t} y_{it} = n_f^p \\ \sum_{i \in I_d^t} y_{it} = n_d^p \end{aligned}\right\} \quad \text{or} \quad \sum_{i \in I_f^t \cup I_d^t} y_{it} = n_{f \cup d}^p, \tag{2}$$

$$y_{it} = 1, \quad \forall i \in I_p^t, \tag{3}$$

$$\sum_{i \in I_{E_f}^t} (1 - y_{it}) \geq n_f^e, \tag{4}$$

$$\sum_{i \in I_{E_d}^t} (1 - y_{it}) \geq n_d^e, \tag{5}$$

$$\sum_{i \in I_{E_g}^t} (1 - y_{it}) \geq n_g^e. \tag{6}$$

Constraints (1) and (2) specify the positional protection requirements for each existing team, as detailed by rules P1 and P2. Specifically, constraint (1) ensures that the required number of goaltenders is protected and constraint (2) enforces the forward and defenseman protection requirements. Note that the constraints shown in (2) are included in the model as a disjunction. That is, one of the constraints on either side of the "or" must be satisfied, but not both. The constraint on the right-hand side of the "or" allows a team that is loaded with defensive talent to protect more than n_d^p defenseman, for example. Constraint (3) enforces the rule that all players with no movement clauses (NMC) must be protected, pertaining to P3. The remaining constraints (4) - (6) enforce the exposure requirements mandated by the expansion draft rules, E1 - E3. Rule P4 was accounted for by simply excluding exempt players from the set I. For the 2017 expansion draft, the specific parameters values can be found in Section 3.2.1.

4.2. Selection Model Constraints

The selection model's (binary) decision variable is x_{it}, defined to be 1 if player $i \in I$ from team $t \in T$ is selected, and 0 otherwise. Let c_{it} be the salary cap hit (in \$M) of player $i \in I$ from team $t \in T$. Let I_f^t, I_d^t, and I_g^t be defined as in the protection model and $I^t = I_f^t \cup I_d^t \cup I_g^t$. Furthermore, let $I_{f_c}^t$, $I_{f_l}^t$, and $I_{f_r}^t$ partition the set of non-exempt forwards from team $t \in T$, I_f^t, into centers, left wingers, and right wingers, respectively. Let $I_{C_f}^t$, $I_{C_d}^t$, and $I_{C_g}^t$ denote the set of non-exempt forwards, defensemen, and goaltenders, respectively, from team $t \in T$ who are are under contract for 2017-18. We also let n_t^s represent the total number of players that must be selected from each team, and n_f^s, n_d^s, n_g^s represent the minimum number of goalies, forwards, and defenseman that

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must be selected by the expansion team. We let $n^s_{C_{f\cup d}}$ and $n^s_{C_g}$ represent the minimum number of skaters and goalies that must be selected that are under contract in the upcoming season. We let ϕ_{min} and ϕ_{max} represent the bounding requirements on the expansion team's aggregate salary. Given solutions to the protection models of all existing teams, we can translate the expansion team roster selection rules into mathematical constraints as follows:

$$\sum_{i\in I^t} x_{it} = n^s_t, \quad \forall t \in T, \tag{7}$$

$$\sum_{t\in T}\sum_{i\in I^t_f} x_{it} \geq n^s_f, \quad \sum_{t\in T}\sum_{i\in I^t_d} x_{it} \geq n^s_d, \quad \sum_{t\in T}\sum_{i\in I^t_g} x_{it} \geq n^s_g, \tag{8}$$

$$\sum_{t\in T}\sum_{i\in I^t_{C_f}\cup I^t_{C_d}} x_{it} \geq n^s_{C_{f\cup d}}, \quad \sum_{t\in T}\sum_{i\in I^t_{C_g}} x_{it} \geq n^s_{C_g}, \tag{9}$$

$$\phi_{min} \leq \sum_{t\in T}\sum_{i\in I^t} c_{it}x_{it} \leq \phi_{max}, \tag{10}$$

$$x_{it} \leq 1 - y^*_{it}, \quad \forall i \in I^t, t \in T. \tag{11}$$

Constraint (7) enforces the number of players that must be selected from each team, corresponding to selection rule S1. Constraint (8) enforces rule S2, the positional requirements for expansion team selections. Constraint (9) ensures the minimum number of skater and goalie selections that must be under contract in the upcoming season, rule S3. Constraint (10) ensures the selected team's salary falls within the allowable window (minimum, ϕ_{min}, and maximum, ϕ_{max}, salary cap in \$M), rule S4. The final constraint, (11), ensures that a player can only be selected by the expansion team if he is not protected by his existing team, corresponding to rule S5. Note that the y^*_{it} term in this constraint is a parameter that comes from the optimal solution to the corresponding protection problem for team t. For the 2017 expansion draft, the specific parameters values for the selection model can be found in Section 3.2.2.

In our computational experiments, we change the goalie constraint in equation (8) to an equality. Since many value metrics are based on time played, and goalies tend to play the entire game, leaving the constraint as an inequality would generally result in many more goalies being chosen than needed. To ensure a balanced forward corps, we added extra constraints to ensure that there would be at least four players selected for each forward position (where the minimum number of center, left-wing, and right-wing players, respectively, is $n^s_{f_c} = n^s_{f_l} = n^s_{f_r} = 4$):

$$\sum_{t\in T}\sum_{I^t_{f_c}} x_{it} \geq n^s_{f_c}, \quad \sum_{t\in T}\sum_{I^t_{f_l}} x_{it} \geq n^s_{f_l}, \quad \sum_{t\in T}\sum_{I^t_{f_r}} x_{it} \geq n^s_{f_r}. \tag{12}$$

Similar constraints can be added to ensure a balance between left and right shooting defensemen, though there seems to be less agreement among coaches on whether every defense pair must be

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balanced. Special teams considerations could be made by identifying certain power play and penalty kill specialists on each team and adding constraints to ensure Vegas selects a certain number of such players. We have omitted such constraints from our model for simplicity.

4.3. Modeling Side Deals

The constraints presented in the previous subsections represent the official expansion draft rules. However, an expansion team can make side deals with existing teams that would result in the promise to select or not select specific players. Such deals can be incorporated in our mathematical optimization framework fairly easily. For example, suppose there is a side deal that leads to the expansion team selecting player i^* from existing team t. This can be modeled by simply setting the corresponding binary selection variable to 1 (i.e., $x_{i^*t} = 1$). In this case, the protection model for team t would not need to be solved. On the other hand, if the side deal requires the expansion team to not select certain players from existing team t, then this can be modeled by setting the protection variable y_{it} for those players to be 0 and the expansion team's selection variable to 0 for the same players. In this case, the protection model would still need to be solved since existing team t will still want to optimize its protections, with the understanding that their players involved in the side deal will not be selected by the expansion team even if they are officially exposed.

4.4. Objective Functions

We model the objective of the existing teams as minimizing a combination of the value of the best asset exposed and the total value of assets exposed in the expansion draft; we expand on what we mean by "value" shortly. For the expansion team, we model their primary objective as maximizing the aggregate value of the team they select. We also include a complementary goal of maximizing financial flexibility, which is an important consideration in the presence of a salary cap. In today's NHL, success is not just about creating a winning team, but also maintaining a winning team by effectively managing salary commitments. Chicago is a notable example of a team that has enjoyed much recent success (Stanley Cup champions in 2010, 2013, 2015), while overhauling large portions of their roster following each championship, due to salary cap constraints.

With the above guiding principles in mind, we define v_{it} to be the value of player i from existing team t. The primary objective function for existing team t will be to minimize the maximum value of all exposed players. This objective function is appropriate when the existing team can lose only one player to the expansion team, which is the situation in the 2017 NHL expansion draft. However, because there may be multiple solutions that all achieve the same optimal value, we add a secondary objective that minimizes the total value of assets exposed for tie-breaking purposes. We combine these two objectives into a single objective by dividing the secondary objective by

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an upper bound β_t, which can be determined for each team based on exposing its most valuable players while still complying with the draft rules:

$$\text{minimize} \max_{\{i:y_{it}=0\}} v_{it} + \frac{1}{\beta_t}\sum_{i\in I^t} v_{it}(1-y_{it}) \tag{13}$$

This objective can be linearized by replacing $\max_{\{i:y_{it}=0\}} v_{it}$ with z and adding the constraint $z \geq v_{it}(1-y_{it})$ for all i. Our notion of "value" is a measure of a player's on-ice contributions and performance, and is not related to salary. We take this approach for simplicity, but acknowledge that financial flexibility is just as valuable for existing teams as it is for the expansion team. For example, an existing team may have an expensive contract that they would like to shed, so they may choose to expose such a player, even if that player's performance value is relatively high.

For the expansion team, the primary objective function is:

$$\text{maximize} \sum_{i\in I^t} v_{it}x_{it} \tag{14}$$

The structure of objective (14) implicitly assumes that the value metric is additive over players. To explore the issue of salary cap flexibility, we consider a complementary objective of minimizing a measure of future salary commitment, which in the context of the 2017 NHL expansion draft we measure as the 2018-19 salary cap hit of the selected players:

$$\text{minimize} \sum_{i\in I^t} \gamma_{it}x_{it} \tag{15}$$

The parameter γ_{it} quantifies the salary cap hit (in \$M) of player i from team t in 2018-19. If a player's contract ends before the start of the 2018-19 season, then $\gamma_{it} = 0$. To explore the trade-off between the two objectives (14) and (15), we construct a composite objective with the salary cap objective weighted by $\alpha \geq 0$:

$$\text{maximize} \sum_{i\in I^t} v_{it}x_{it} - \alpha \sum_{i\in I^t} \gamma_{it}x_{it} \tag{16}$$

Note the minus sign in objective (16) is needed to convert the salary cap flexibility objective from a minimization to a maximization.

4.5. Complete models

Here, we summarize the complete protection and selection models used in the computational experiments. The protection model for each existing team t is

$$\begin{aligned} \underset{y_{it}}{\text{minimize}} &\quad (13) \\ \text{subject to} &\quad (1)-(6) \end{aligned}$$

and the selection model for the expansion team is

$$\begin{aligned} \underset{x_{it}}{\text{minimize}} &\quad (16) \\ \text{subject to} &\quad (7)-(12). \end{aligned}$$

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4.6. Data

For our value metric, v_{it}, we used 2016-17 Point Shares (PS) values (Kubatko (2011)). PS is "designed to estimate a player's contributions in terms of points" in the standings. The metric was motivated by the work done by Bill James to estimate player contributions to wins in baseball (James and Henzler (2002)). To give an idea about PS for those who are unfamiliar with it, for the 2016-17 season, the range of PS values amongst players that played at least 10 games in 2016-17 was -1.2 to 16.0, with the 10th and 90th percentiles being 0.2 and 7.8 (see Appendix). The top five teams in the regular season standings averaged a total PS value of 114.6, while the bottom five averaged a total PS value of 64.2.

We emphasize that PS was chosen as an illustrative example of the type of metric that can be used by our model, but by no means is it the only one. Our framework is general enough to accommodate any statistic, as long as it can be applied to players of all positions (forwards, defensemen, and goalies). Of course, using a metric based on historical performance may lead to overestimating or underestimating the value of certain players. For example, existing teams might want to expose a high-value player if that player is older or has an expensive contract. Similarly, the expansion team might be interested in a promising young player whose performance to date has not reflected their potential. Teams that have internal valuations for players that take into account other considerations beyond past performance can use those in place of PS in v_{it}. Another point to note is that the computational results presented later in this paper are dependent on the particular choice of value metric we use. Thus, we caution the reader from reading too much into the specific PS values, but rather focus on the broader takeaways from our analyses.

We obtained PS data from hockeyabstract.com. We validated PS against team performance in the Appendix, which shows that the sum of PS over all players on a team is well-correlated to team performance in the standings over the last three regular seasons. We obtained salary and contract information from capfriendly.com. This information was recorded at three different time points (November 30, 2016, March 1, 2017, and June 17, 2017) to capture trades or contract signings that would affect the protection and selection models. We also scoured news reports and official trades registered by the NHL around the time of the expansion draft to capture all recorded side deals that Vegas made with existing teams. The inclusion of these side deals essentially constitutes a fourth time point for analysis.

Finally, we comment on our approach to dealing with free agents. Generally, we removed pending unrestricted free agents (UFAs) to avoid them being protected/selected. The exception was when a free agent was needed for model feasibility. For example, for the March 2017 snapshot, Calgary's protection problem would not be feasible if Brian Elliott, a pending UFA goalie, was excluded.

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The reason was that one of their other goalies, Chad Johnson, was also a UFA leaving only a single goalie, Tom McCollum, to satisfy both protection and exposure requirements. Thus, between March 2017 and the expansion draft, Calgary would need to make some changes to their goalie roster to satisfy the requirements. For simplicity, we assume all restricted free agents (RFAs) are given a qualifying offer by their existing team, and thus eligible for protection/selection.

4.7. Implementation

All experiments were implemented in C++ on an 8-core Intel Core i7-2670QM processor machine with 8GB of RAM running Ubuntu 14.04 LTS. We used the CPLEX mixed-integer programming solver from the IBM ILOG CPLEX Optimization Studio version 12.6.3 with default search settings for all experiments.

The disjunction in constraint (2) in the protection model is implemented using indicator constraints `IloOr` and `IloAnd` within CPLEX. Such constraints offer an alternative to "big-M" formulations, which often suffer from numerical instability (Bonami, Lodi, Tramontani, and Wiese 2015). Given an indicator constraint, the solver elects to: i) use MIP pre-processing to derive a tighter, more stable disjunctive constant, or ii) branch on the indicator constraint itself. In the latter case, the solver partitions the search space based on the logical condition of the disjunction. For the `IloOr` used to model constraint (2), the solver partitions the search space into two subspaces: one where the existing team chooses to protect 7 forwards and 3 defenseman, and one where the existing team chooses to protect 8 total skaters. For the protection model objective function, Eqn. (13), we set each β_t equal to 10,000, which is sufficiently large to separate the primary and secondary objectives in the protection model since individual Point Shares values are typically in the single and low double digits (see Appendix). Solving 30 instances of the protection model (including the partition induced by the disjunction) and one instance of the selection model takes about three seconds total.

5. Results

In this section, we present results from computational experiments aimed at exploring our modeling framework. The first subsection examines the baseline solutions from our models regarding protections and selections at our four time points. These results show how Vegas' potential team value changed over the course of the season. The second subsection details the trade-off between Vegas' potential performance and salary cap flexibility; as the desire for financial flexibility increases, the performance of the team decreases. The third subsection focuses on our "collaboration" experiment, which provides an indication of how much Vegas' team value could have been degraded if

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teams consciously aimed to reduce Vegas' team value. The fourth subsection describes our "leverage" experiment, quantifying the impact to Vegas by not selecting the optimal player suggested by the model. The last subsection compares the protection and selection decisions produced by our models based on the June 17, 2017 snapshot to the actual decisions that occurred a few days later.

5.1. Value of Vegas' team over time

We consider the projected quality of Vegas' team at four time points in 2016-17: November 30, 2016 (early in the regular season), March 1, 2017 (immediately following the trade deadline), and two alternatives for June 17, 2017 (days prior to the expansion draft), as illustrated in Figure 1. The difference between the two June 17, 2017 snapshots is that one excludes the impact of the side deals (snapshot #3) while the other one includes the side deals (snapshot #4). In essence, snapshot #3 represents what was known publicly at that time, whereas snapshot #4 represents what Vegas' knew internally. Because we are interested in measuring the maximum team value Vegas could achieve at each time point, we set $\alpha = 0$ in the objective function.

November 30, 2016	March 1, 2017	June 17, 2017	June 17, 2017
Snapshot #1	**Snapshot #2**	**Snapshot #3**	**Snapshot #4**
Point Shares: 120.0	Point Shares: 110.6	Point Shares: 103.7	Point Shares: 82.4
Cap Hit: \$72.1M	Cap Hit: \$70.9M	Cap Hit: \$72.6M	Cap Hit: \$68.9M
		(excluding side deals)	*(including side deals)*

Figure 1 Maximum value of Vegas' team over 2016-17 season

As shown in Figure 1, the maximum value of Vegas' potential team degrades as the season progressed, with the total salary staying roughly constant. As expected, leading up to draft day teams made adjustments that ultimately weakened the best possible roster Vegas could produce. In particular, Vegas lost 13.6% in potential value from November 2016 to June 2017 (excluding side deals), which translates to roughly 12.3 points in the standings (see Figure 5 in Appendix). If we account for the side deals that Vegas made, its maximum team value is further degraded, as shown in the final snapshot. This further degradation is due to Vegas making deals to not select the best exposed players from each team, which allows them to negotiate for additional assets in the form of draft picks and prospects. More discussion on this point is provided in Section 5.5.

5.2. Balancing performance and financial flexibility

Next, we examine the interplay between PS and future salary cap flexibility by varying the weight α in the objective function (16). The baseline team from the June 17, 2017 snapshot excluding side deals had \$31.67M in salary commitments for 2018-19. As salary cap flexibility is increased (i.e.,

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fewer salary commitments for 2018-19), the PS of the associated roster goes down, as illustrated in Figure 2. A team with maximum salary cap flexibility (i.e., zero salary commitments for 2018-19) turns out to have 14.9% lower value than the baseline team, which does not optimize for salary cap flexibility at all and purely optimizes for team PS. On the other hand, increasing future salary cap commitments increases team PS, up to where it will level off at the value of the best team that can be constructed, given the exposed players. The shape of the curve is nonlinear and suggests that a reasonable balance can be struck between these two objectives, which would buy Vegas financial flexibility without seriously degrading potential team performance. For example, Vegas can more than halve its 2018-19 salary commitments with only a 4.9% decrease in team value.

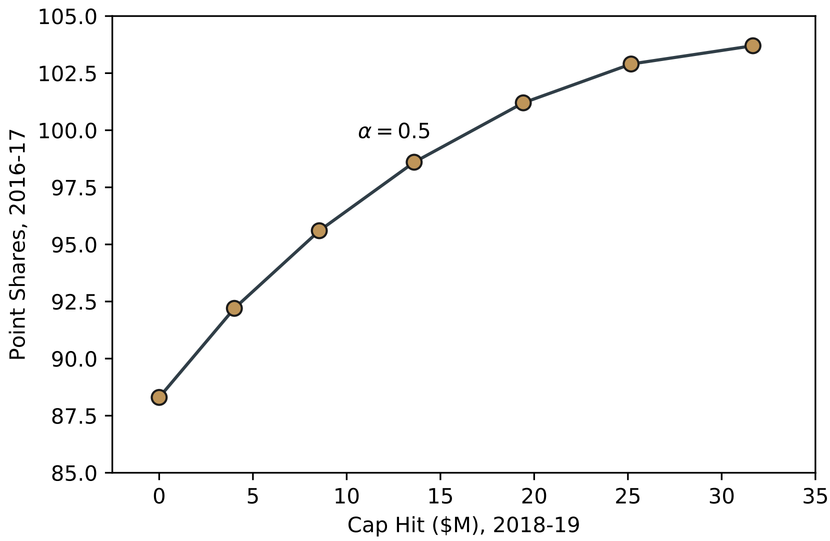

Figure 2 Vegas' team value (in 2016-17 Point Shares) as a function of future salary cap flexibility (in 2018-19 expenditures).

5.3. Collaboration

In this experiment, we model the possibility of existing teams making trades, or "collaborating," in an effort to position themselves better than they would be able to independently. Collaborating teams essentially combine their protection limits and improve the value of assets protected. The goal of this section is to demonstrate that our models can be used to explore an interesting but challenging combinatorial question that arises in the context of an expansion draft using a mathematically tractable framework. While clearly an approximation to a realistic situation, our approach does provide an estimate on the possible impact of collaboration by modeling "fluid" player transactions between teams.

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In this section, we consider all pairwise collaborations by forming, for any two teams t_1 and t_2, a joint protection model where all the limits are doubled and the two rosters are merged into one larger roster. Instead of all 30 teams solving their protection models independently and passing the results to the Vegas selection problem, we now solve the two-team joint protection model for t_1 and t_2 plus the remaining 28 individual protection models. We then solve the Vegas selection problem for each of these 435 pairwise combinations and record the degradation in Vegas' PS. For the collaborating teams, we combine their objective functions so they are minimizing the maximum value player exposed for each team.

Next, we assess the impact of larger collaborations (i.e., k-teams collaborations for $k > 2$ and k even). As with the pairwise experiments, we pool players from k teams together and scale the protection model limits (e.g., positional requirements) accordingly. Due to the combinatorial explosion associated with finding the optimal k-team collaboration, we greedily order the previously assessed pairwise collaborations in increasing order of resulting Vegas PS, identifying the most effective distinct pairwise collaboration. We then evaluate the k-team collaboration problem by assuming the top $k/2$ pairs are all collaborating together, and again record the PS of the resulting Vegas team. This heuristic approach does not guarantee the optimal k-team collaboration, but serves as a reasonable estimate. Finally, we consider a "mega-collaboration" model, where all 30 teams solve a single protection model with 30 times the protection limits and a single roster composed of all 30 teams' players. This mega-collaboration model provides an estimate of the worst-case impact of collaboration on Vegas, modeling the situation where trades between teams are fluid and made with the sole purpose of minimizing exposed value to Vegas.

The results from the collaboration experiment are shown in Figure 3. The y-axis measures the PS value of Vegas' roster and the x-axis measures the degree of collaboration in terms of the number of teams involved. As more teams are included in the collaboration model, Vegas' team value degrades further. In the case that all 30 teams are working together, we see that Vegas' overall team value drops significantly, by roughly 34%, going from a team PS value of 103.7 to 68.6.

Our results from this experiment suggest that trades and other personnel decisions made by the existing teams can have a significant impact on the quality of the drafted expansion team.

5.4. Leverage

In this experiment, we use our optimization framework to investigate the possible value that Vegas can extract from an existing team by agreeing not to select the optimal player suggested by the model. Given that expansion teams are typically building for the future, Vegas can "leverage" this potential advantage to extract additional assets such as draft picks or prospects from existing teams.

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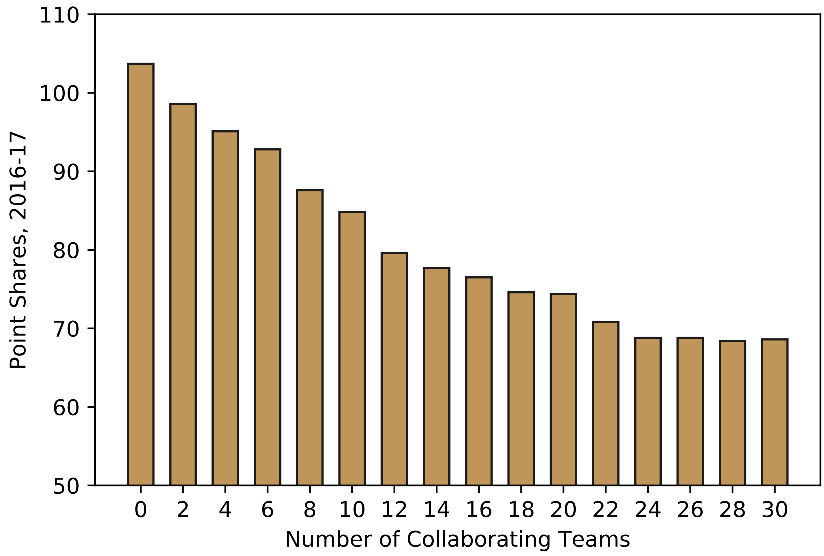

Figure 3 The effect of existing team collaboration on Vegas' PS value.

Building off of the baseline results from the June 17, 2017 snapshot (excluding side deals), we modify the selection model so that, for each existing team, Vegas is prevented from picking the player it previously chose. This step is accomplished by simply setting the appropriate binary variable to 0 in the selection model. We model this scenario for each team independently, so the selection problem is re-solved 30 times. Each time, we record the reduction in Vegas' total team value after optimizing its selections compared to the baseline solution, as well as the change in value of the player lost by each existing team.

As we see from Figure 4, some teams, such as Florida (FLA), Pittsburgh (PIT), and Dallas (DAL), stand to gain a lot from being able to protect an additional player. These are teams that, in the baseline model, had goalies (high value players) selected by Vegas. As such, Vegas should be able to command more in return for not selecting these players. Chicago (CHI) and Colorado (COL) also see net benefit by entering a leverage agreement, though to a lesser degree. On the other extreme, Carolina (CAR) originally had a high-value forward selected by Vegas. However, if Vegas does not select this player, Vegas would instead select one of Carolina's goalies with even higher value, and thus Carolina would lose even more value from its roster. This is an interesting example where Vegas' ability to re-optimize its selections results in Carolina losing value, since the selection model is trying to mitigate Vegas' loss from not being able to choose the originally selected forward from Carolina. It illustrates that there is some risk in an existing team entering into an agreement with the expansion team; they are not guaranteed to be better off if the expansion team agrees not to select the currently preferred player. Additionally, this Carolina example suggests that an existing team could expose a higher value player if they know the expansion team will not select

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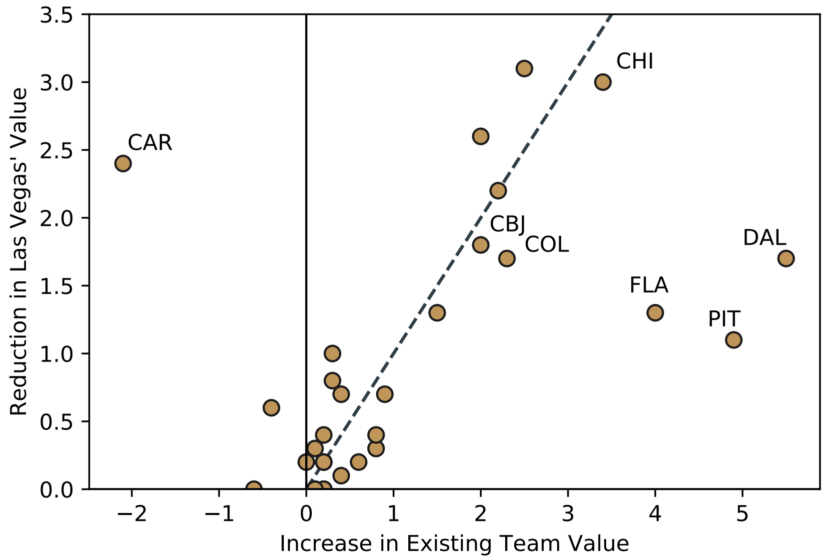

Figure 4 Leverage experiment: with a small reduction in Vegas' potential team value, existing teams can increase their own post-draft value. Teams to the right of the dashed line increase their own value more than Vegas loses value, and those to the left do not. Both axis values in 2016-17 Point Shares.

them, and instead protect a lower-value player at another position that is of more relative value to the expansion team. Unfortunately for Carolina in this situation, even if they knew that Vegas would not take their goaltender, the expansion draft rules would prevent them from exposing the goaltender in favor of a skater. However, if an existing team knows that the expansion team is uninterested in a specific forward or defenseman, they could expose this player and protect a lower value skater at the other position by choosing the 8 skaters and 1 goalie protection configuration. This scenario could be modeled by simply setting the higher value player's associated binary variable in the protection model to 0.

For the top 10 teams that would gain the most from Vegas not selecting the optimal player suggested by the model, Vegas' team value only degrades by 1.9% on average. This experiment can also help explain some of Vegas' actual side deals. Columbus (CBJ), for example, gave Vegas a first and second-round draft pick to ensure Vegas selected William Karlsson instead of Boone Jenner. Our baseline models have Columbus exposing Boone Jenner and Vegas selecting him. But if Jenner is protected and Karlsson is selected instead, this leverage experiment shows that Columbus' team value increases by two points. Overall, we see that Vegas does not actually lose much in value by forgoing the optimal player as suggested by the model from an existing team. If the existing team does not realize this fact, then Vegas can potentially extract significant assets from those teams without actually giving much up.

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5.5. What actually happened

With regards to protection decisions, the principle of minimizing exposed value played out as expected for most of the existing teams. Model protections for Anaheim, Edmonton, Los Angeles and Tampa Bay matched perfectly with reality. Furthermore, Winnipeg, Minnesota, Columbus and the New York Islanders made trades to protect exposed players that were protected by our model, and so in effect the model also matched reality. The model was off by one decision for seven teams (the model protected Eric Fehr for the Maple Leafs over Matt Martin, and James Neal for the Nashville Predators over Calle Jarnkrok, to give a few examples). The model solution and actual decisions differed by at most two players for 21 of the teams. Four teams deviated from the model protections by four decisions, one of which was the Florida Panthers, who elected to use the eight-skater protection configuration (our model used the seven forward, three defensemen configuration) and curiously arranged to not protect Jonathan Marchessault (a promising young player) and make him available for Vegas to select if Vegas also agreed to acquire Reilly Smith, whose contract Florida wanted to unload. Overall, for 22 of the 30 teams, our model's optimal protection configuration (i.e., seven forwards and three defense men versus eight skaters) matched the actual configuration used by the team. Differences between the model results and reality often arose when a player's 2016-17 PS value was not representative of their current/future ability, leading to a player being exposed by our model when in reality he was protected by his team. This scenario occurred for both young, up-and-coming players like Curtis Lazar and Valeri Nichushkin, as well as for established players who had a poor season such as Tyler Ennis and Erik Gudbranson. Arguably, other factors like age played a role in those decisions, but that information was not captured in our model.

Larger discrepancies were observed between our baseline selection model results (snapshot #3) and Vegas' selection decisions. These discrepancies were largely due to the fact that Vegas made many side deals with teams to select or not select certain players for additional considerations such as draft picks and prospects. For example, Travis Hamonic (New York Islanders), Jonas Brodin (Minnesota), and Josh Anderson (Columbus) were all exposed by their respective teams, but those teams gave Vegas draft picks to ensure they were not selected. As mentioned above, Jonathon Marchessault (Florida), a valuable young player, was purposely exposed and selected by Vegas in return for Vegas also acquiring his teammate Reilly Smith, who had an expensive contract. The deals played a significant role in Vegas' selection decisions. Adding these side deals improved the model's performance with respect to what actually happened. Overall, these side deals resulted in Vegas acquiring one additional active player (Reilly Smith), two inactive players (David Clarkson and Mikhail Grabovski), three prospects, and a total of eight extra draft picks. Vegas' 2016-17 PS total and 2017-18 cap hit for their drafted players were 72.1 and $56.0M respectively. Taking into

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account all assets acquired on draft day, these values increase to 75.2 and $66.3M. The PS value of the actually selected team is still lower than the optimal value according to our model in the side deal snapshot, but accounting for side deals reduced the over-projection of the optimization model significantly.

The impact of side deals can be measured with our models if all the players involved are on NHL rosters already. However, since the compensation for making a side deal is typically some combination of draft picks and/or prospects, the overall value of the expansion team degrades, when measured in terms of the official selections (compare the value of snapshot #3 and #4 in Figure 1). Although the value of prospects or draft picks are not explicitly considered in our model, they could potentially be included in the v_{it} value of the player the expansion team selects, to account for the fact that they are getting more value through these additional assets. Of course, qualitative evaluation is always an important part of analyzing players with limited professional data, such as prospects in a farm system. In this case, qualitative assessment refers to assessment outside of the model but still potentially based on statistics from amateur leagues.

Finally, revisiting our financial flexibility analysis from Section 5.2, we note that Vegas' actual selections resulted in a 2018-19 cap hit of $18.1M. By virtue of being quite far from the efficient frontier illustrated in Figure 2, it would seem that Vegas had additional considerations or alternate player valuation metrics not considered in our model.

6. Conclusion

In this paper, we developed a novel and tractable optimization framework that can serve as a decision-support tool in expansion draft optimization, for both the existing teams and the expansion team. Applied to the 2017 NHL expansion, we were able to generate protection lists for the 30 existing teams and the selection list for Vegas in about three seconds, where both sides are optimizing to maximize the value of their assets as measured by the Point Shares metric. We demonstrated the application of our framework by measuring Vegas' team value under several different scenarios.

By applying our modeling framework to data from different time points over the course of the 2016-17 season, we showed that the total potential value of Vegas' team was degraded by over 13% due to the actual roster moves and signings of the other teams. We demonstrated how our optimization approach can balance performance with financial flexibility when constructing a team. For example, a financially flexible team could reduce 2018-19 salary commitments by over 50% while only sacrificing 4.9% in performance. In the case of hypothetical "collaboration" between existing teams – personnel moves that would enable existing teams to better protect their current assets – Vegas could stand to lose upwards of 34% in total PS. On the flip side, our framework can

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identify teams that Vegas might have leverage over in terms of its ability to negotiate side deals that avoid selecting a particular player, while having minimal impact on Vegas' overall team value.

Although our framework was a simplification of the complicated decision-making processes that surround the expansion draft, and despite the model's lack of access to domain-specific information like side-deals between Vegas and other teams, our optimization results were reasonably well-aligned with reality. We believe such an optimization framework can serve as a foundation for future expansion draft optimization research and decision-making, especially since it is general and flexible enough to incorporate the rule variations typically found in expansion drafts in other sports. Future research may also consider how uncertainty in the player value metric, v_{it}, can be incorporated into the model, especially if the data represents projections on future performance. Stochastic or robust optimization approaches are natural candidates.

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7. Point Shares background

Here, we provide some additional insight into the PS metric. First, we show in Figure 5 that team PS is a relatively good surrogate for overall team performance, using regular season team points and team PS for all teams from 2014-15 to 2016-17.

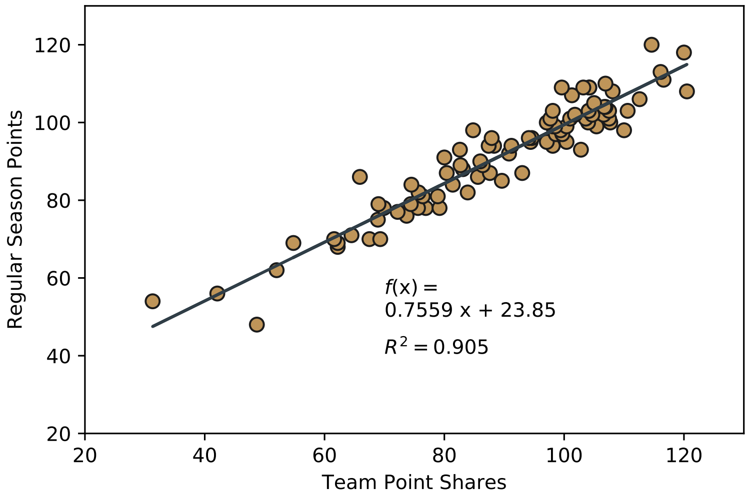

Figure 5 The relationship between regular season team points and team PS.

To get a sense of the typical range of PS values for a player, Figure 6 shows the distribution of player PS for 2016-17.

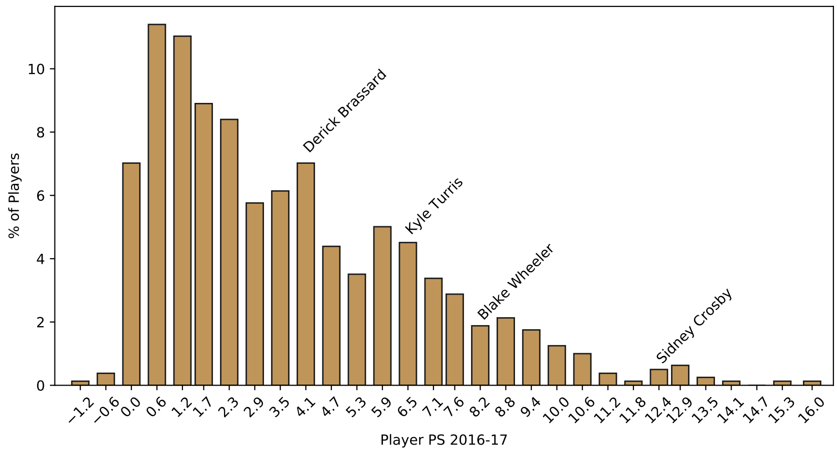

Figure 6 Point Share distribution for 2016-17 players with at least ten games played. Min = -1.2, max = 16.0, 10th percentile = 0.2, 90th percentile = 7.8

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